GEOPHYSICS, VOL. 61, NO. 1 (JANUARY-FEBRUARY 1996); P. 202-210, 6 FIGS., 2 TABLES.

Removal of surface-related wave phenomena—The marine case

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ABSTRACT

Removal of the effects of the free surface from seismic reflection data is an essential preprocessing step before prestack migration. The problem can be formulated by means of Rayleigh's reciprocity theorem which leads to an integral equation of the second kind for the desired pressure field that does not include these free-surface effects. This integral equation can be solved numerically, both in the spatial domain and in the double Radon domain. Solving the integral equation in the double Radon domain has the advantage of reducing the computation time significantly since the kernel of the integral equation becomes dominant diagonally. Two methods are proposed to solve the integral equation: direct matrix inversion and a recursive subtraction of the free-surface multiples using a Neumann series. Both methods have been developed and tested on a synthetic data set, which was computed with the help of an independent forward-modeling scheme.

INTRODUCTION

Surface-related multiples are a classic problem in marine seismic data processing. The problem is especially severe in areas where the water bottom has a high velocity contrast: the multiples tend to decay slowly and, because the energy is trapped in the water layer, the effect of the free surface degrades the quality of the seismogram significantly. Several methods have been developed to attack the problem of the free surface, such as predictive deconvolution that makes use of the fact that the multiples appear in the data with certain periodicity (Robinson and Treitel, 1980), and methods based on moveout, which make use of the assumption that the surface-related multiples are low-velocity events with respect to the primaries. These methods are often ineffective, because of the assumptions that are inherent in conventional multiple attenuation schemes, which is the reason that new attention has been paid to the problem of the free surface.

An effective method to remove the effects of the free surface should require no a priori information, neither structural nor material, about the subsurface geology, and must leave any relevant subsurface information present in the data unaffected. Verschuur et al. (1992) describe a method that is based on wave theory. They derive an expression for the multiple contaminated upgoing pressure wavefield as a function of the multiple free upgoing pressure wavefield as the subsurface impulse response. The removal of the surface-related multiples is performed by calculating the inverse operator such that an expression for the upgoing wavefield excluding the free-surface effects is obtained. In addition, an estimation of the wavelet is made based on some minimum energy criterion. Carvalho and Weglein (1992) propose a similar method based on an inverse scattering series solution method that removes the surface-related multiples without the need to separate the wavefield into its up- and downgoing constituents. In the scattering series, each term in the series removes a higher-order surface multiple.

In this paper, we present a method to remove the surface-related wave phenomena that is based on Rayleigh's reciprocity theorem, which formulates the interaction of two nonidentical states in a domain. One state is identified with the actual situation where the free surface is present, while the other state is the desired one that differs only from the actual state by the absence of the free surface. Using Rayleigh's reciprocity theorem, an integral equation can be derived in which the desired pressure wavefield in an unbounded medium, where no free-surface effects are present, is expressed as a function of the actual measured pressure in a bounded medium that incorporates the free-surface effects. The integral equation can be solved numerically by direct matrix inversion or iteratively by a Neumann series. Using the integral formulation, we prove that the Neumann iteration series is convergent for any geology in the time domain and can therefore be considered as a legitimate alternative to solve the integral equation. Further-

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more, the integral formulation enables us to transform the integral equation to the double Radon domain. The main advantage of solving the integral equation in the double Radon domain is that the computation time can be reduced significantly since the kernel of the integral equation becomes diagonally dominant.

RAYLEIGH'S RECIPROCITY THEOREM

We start the analysis with the acoustic equations in the space-frequency domain:

$$\partial_k \hat{p}(\mathbf{x}) + j\omega\rho(\mathbf{x})\hat{v}_k(\mathbf{x}) = \hat{f}_k(\mathbf{x}), \quad (1)$$

$$\partial_k \hat{v}_k(\mathbf{x}) + j\omega\kappa(\mathbf{x})\hat{p}(\mathbf{x}) = \hat{q}(\mathbf{x}), \quad (2)$$

in which

$\hat{p}(\mathbf{x})$ = acoustic pressure [Pa],
$\hat{v}_k(\mathbf{x})$ = particle velocity [m s^{-1}],
$\rho(\mathbf{x})$ = volume density of mass [kg m^{-3}],
$\kappa(\mathbf{x})$ = compressibility [Pa^{-1}],
$\hat{q}(\mathbf{x})$ = volume density of volume injection rate [s^{-1}],
$\hat{f}_k(\mathbf{x})$ = volume density of volume force [N m^{-3}].

We have used a temporal Fourier transformation with time factor exp $(j\omega t)$ to transform the time-domain acoustic field quantities $\{p, v_k\}$ to the frequency-domain counterparts $\{\hat{p}, \hat{v}_k\}$. Further, $\mathbf{x} = \mathbf{i}_1 x_1 + \mathbf{i}_2 x_2 + \mathbf{i}_3 x_3$ denotes the 3-D position vector in the right-handed orthogonal Cartesian reference frame with origin O.

A reciprocity relation interrelates the field quantities that are associated with two nonidentical physical states that could occur in the same time-invariant domain in space. We consider a bounded domain D with a boundary surface ∂D. The acoustic state in a domain D is a composition of three states:

1) the field state, described by $\hat{p}$ and $\hat{v}_k$;
2) the material state, described by ρ and κ; and
3) the source state, described by $\hat{f}_k$ and $\hat{q}$.

The two physical states are distinguished by superscripts A and B. The global form of the reciprocity relation, applied to a domain D with boundary ∂D, that interrelates the two states A and B is given by

$$\int_{\mathbf{x}\in\partial D} (\hat{p}^A\hat{v}_k^B - \hat{p}^B\hat{v}_k^A)\nu_k dA$$

$$= \int_{\mathbf{x}\in D} [j\omega(\rho^B - \rho^A)\hat{v}_k^A\hat{v}_k^B - j\omega(\kappa^B - \kappa^A)\hat{p}^A\hat{p}^B]\, dV$$

$$+ \int_{\mathbf{x}\in D} (\hat{f}_k^A\hat{v}_k^B + \hat{q}^B\hat{p}^A - \hat{f}_k^B\hat{v}_k^A - \hat{q}^A\hat{p}^B)\, dV,$$

in which ν_k denotes the unit vector normal to ∂D pointing away from D (see Figure 1). In the following section, the two states are defined that are to be interrelated to remove the effect of the free surface.

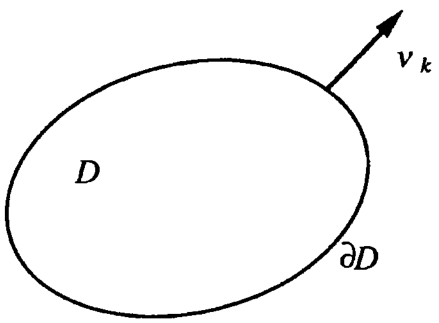

FIG. 1. Domain of application for the reciprocity theorem.

DEFINITION OF THE TWO STATES

In the marine case, we have the situation shown in Figure 2. The domain of interest is the half-space $D = (\mathbf{x} \in R^3 | -\infty < x_1, x_2 < \infty, 0 < x_3 < \infty)$, which can be divided into the water layer D_w and the earth geology D_g with boundary ∂D_g. The material constants in D_w are $\{\rho_w, \kappa_w\}$, and the material constants in D_g are $\{\rho_g, \kappa_g\}$. State A is taken as the (known) actual marine configuration. An impulsive point source located at position $\mathbf{x}^S$ below the water surface $x_3 = 0$ generates the acoustic waves. Let this wavefield be denoted as $\{\hat{p}^A, \hat{v}_k^A\} = \{\hat{p}, \hat{v}_k\}(\mathbf{x}|\mathbf{x}^S)$. The wave speed in the water layer is c_w. The spectrum of volume injection is $\hat{q}^S = \hat{q}^S(\omega)$ and is assumed to be known. The volume density of body force is set to zero. The seismic response is measured by point receivers at position $\mathbf{x}^R$. In Figure 2, $x_{3,min}$ denotes the top of the geology. In state B, the desired state, the water layer extends to x_3 3 $-\infty$ (see Figure 3). In this case, the plane defined by $x_3 = 0$ is just an artificial boundary. We choose a point source with an identical spectrum to the one in state A located at the receiver

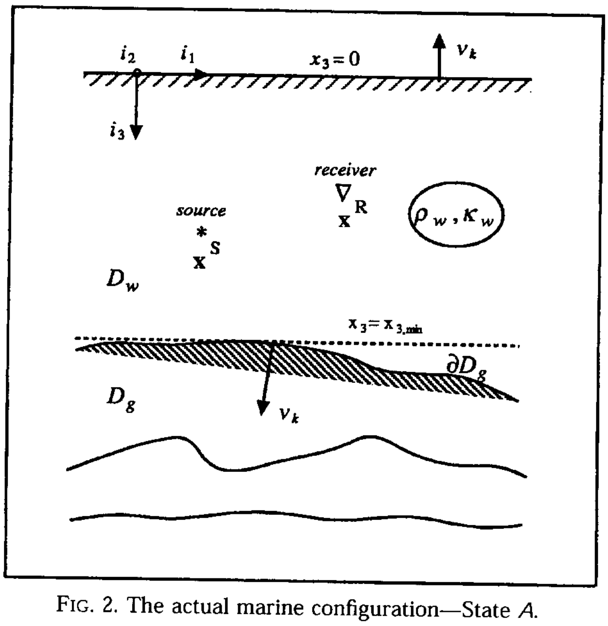

FIG. 2. The actual marine configuration—State A.

position $\mathbf{x}^R$ (see Table 1). This wavefield is denoted as $\{\hat{p}^B, \hat{v}_k^B\} = \{\hat{p}^d, \hat{v}_k^d\}(\mathbf{x}|\mathbf{x}^R)$. It is noted that $\hat{p}(\mathbf{x}^R|\mathbf{x}^S)$ indicates the pressure wavefield measured at receiver position $\mathbf{x}^R$ due to a source located at position $\mathbf{x}^S$.

DERIVATION OF THE INTEGRAL EQUATION

With the states mentioned above, the reciprocity theorem is applied to domain $D_w \cup D_g$, enclosed by the boundary at the water surface $x_3 = 0$ and a semi-infinite sphere S_Δ of radius Δ with its center at the origin O, and where the limit Δ 3 ∞ is considered (see Figure 4). We then arrive at

$$\int_{(x_1,x_2)\in R^2} \hat{p}^d(x_1, x_2, 0|\mathbf{x}^R)\hat{v}_3(x_1, x_2, 0|\mathbf{x}^S)\, dA = \hat{q}^S\hat{p}(\mathbf{x}^R|\mathbf{x}^S) - \hat{q}^S\hat{p}^d(\mathbf{x}^S|\mathbf{x}^R), \quad (4)$$

where we have taken into account that the pressure field of the actual situation vanishes at $x_3 = 0$. The contribution of the boundary integral over the semi-sphere is also zero. This can be seen as follows: It can always be assumed that outside some sphere of bounded radius the fluid is homogeneous with the material constants $\{\rho_w, \kappa_w\}$ (see Figure 4). Taking the acoustic wavefield in both states to be causally related to the action of their sources, for large Δ we can use the far-field approximations of the radiated fields on S_Δ. As a consequence, the contribution of the surface integration over S_Δ vanishes when Δ 3 ∞. Equation (4) is an integral equation for the desired pressure wavefield $\hat{p}^d$.

For the acoustic pressure, we define the 2-D Fourier transform pair of the Radon type with respect to the horizontal receiver coordinates, given by

$$\tilde{p}(p_1, p_2, x_3|\mathbf{x}^S) = F^R\{\hat{p}(x_1, x_2, x_3|\mathbf{x}^S)\} = \int_{(x_1,x_2)\in R^2} \hat{p}(x_1, x_2, x_3|\mathbf{x}^S) \times \exp(j\omega p_1 x_1 + j\omega p_2 x_2)\, dA \quad (5)$$

and

$$\hat{p}(x_1, x_2, x_3|\mathbf{x}^S) = F^R_{-1}\{\tilde{p}(p_1, p_2, x_3|\mathbf{x}^S)\} = \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}(p_1, p_2, x_3|\mathbf{x}^S) \times \exp(-j\omega p_1 x_1 - j\omega p_2 x_2)\, dA; \quad (6)$$

and the 2-D Fourier transform pair of the Radon type with respect to the horizontal source coordinates, given by

$$\tilde{p}(\mathbf{x}^R|p_1, p_2, x_3) = F^S\{\hat{p}(x^R|x_1, x_2, x_3)\} = \int_{(x_1,x_2)\in R^2} \hat{p}(\mathbf{x}^R|x_1, x_2, x_3) \times \exp(-j\omega p_1 x_1 - j\omega p_2 x_2)\, dA \quad (7)$$

and

$$\hat{p}(\mathbf{x}^R|x_1, x_2, x_3) = F^S_{-1}\{\tilde{p}(\mathbf{x}^R|p_1, p_2, x_3)\} = \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}(\mathbf{x}^R|p_1, p_2, x_3)$$

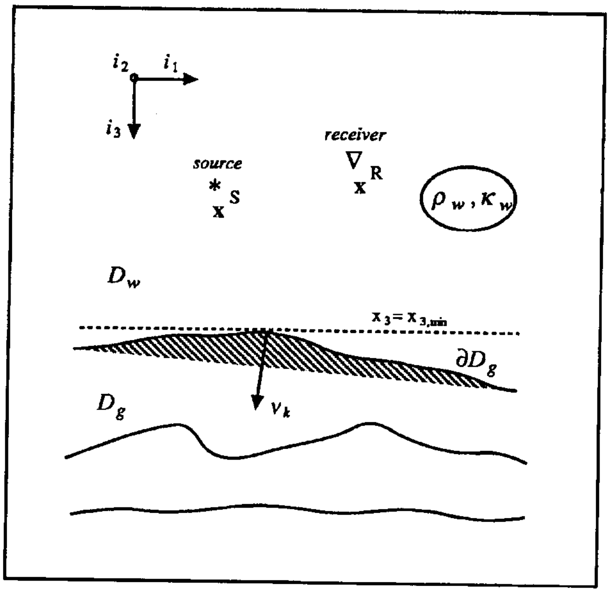

FIG. 3. The desired marine configuration—State *B*.

Table 1. Reciprocity between the actual wavefield and the desired wavefield.

	State A	State B
Field state	$\{\hat{p}, \hat{v}_k\}(\mathbf{x}\|\mathbf{x}^S, \omega)$	$\{\hat{p}^d, \hat{v}_k^d\}(\mathbf{x}\|\mathbf{x}^R, \omega)$
Material state	$\{\rho_w, \kappa_w\}$ in D_w $\{\rho_g, \kappa_g\}$ in D_g	$\{\rho_w, \kappa_w\}$ in D_w $\{\rho_g, \kappa_g\}$ in D_g
Source state	$\{\hat{q}^S(\omega)\delta(\mathbf{x} - \mathbf{x}^S), 0\}$	$\{\hat{q}^S(\omega)\delta(\mathbf{x} - \mathbf{x}^R), 0\}$
	Domain $D_w \cup D_g$	

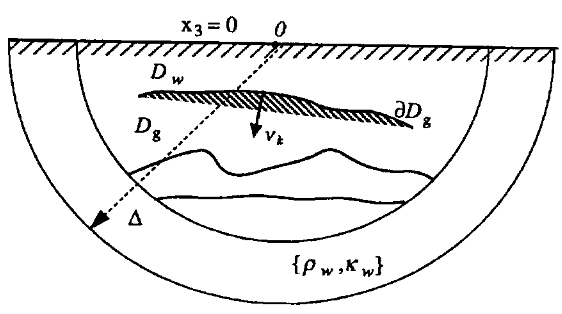

FIG. 4. Configuration for the derivation of the integral equation.

$$\times \exp(j\omega p_1 x_1 + j\omega p_2 x_2)\, dA; \qquad (8)$$

where the horizontal slownesses are described by p_1 and p_2. Similar definitions apply for the Fourier transforms of the particle velocity $\hat{v}_k$.

Application of these transforms to the field quantities in the left-hand side of equation (4) leads to

$$\left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1, p_2)\in R^2} \bar{p}^d(\mathbf{x}^R|p_1, p_2, 0)\bar{v}_3(p_1, p_2, 0|\mathbf{x}^S)\, dA = \hat{q}^S\hat{p}(\mathbf{x}^R|\mathbf{x}^S) - \hat{q}^S\hat{p}^d(\mathbf{x}^R|\mathbf{x}^S). \quad (9)$$

Note that we applied physical reciprocity in order to interchange the horizontal source and receiver dependences of the desired pressure wavefield. This follows directly from the reciprocity relation [see equation (3)] taking for states A and B the desired state with interchanged source and receiver positions. In the next section, an expression is derived for the particle velocity $\bar{v}_3$ in terms of the measured pressure wavefield.

AUXILIARY RELATION FOR THE PARTICLE VELOCITY

In the preceding section, an integral equation was derived for the desired pressure wavefield. In this integral equation [see equation (9)], the particle velocity of the actual state occurs. In this section, an expression is derived for the velocity field in terms of the measured pressure wavefield.

We start with the application of the reciprocity theorem to domain D', defined as $D' = (\mathbf{x} \in R^3| -\infty < x_1, x_2 < \infty, 0 < x_3 < x_3^R)$. State A is the actual field. State B is an auxiliary source-free wavefield with a pressure field that is set to zero at level $x_3 = x_3^R$ (see Table 2) by taking $P_w(\mathbf{x}) = \exp(j\omega p_1 x_1 + j\omega p_2 x_2)\sin(\omega\Gamma_w(x_3^R - x_3))$, in which

$$\Gamma_w = \left(\frac{1}{c^2} - p_1^2 - p_2^2\right)^{1/2} \qquad (10)$$

denotes the vertical slowness. Reciprocity between the two states yields

Table 2. Reciprocity between the actual wavefield and an auxiliary source-free wavefield.

	State A	State B
Field state	$\{\hat{p}, \hat{v}_k\}(\mathbf{x}\|\mathbf{x}^S, \omega)$	$\left\{1, \frac{-\partial_k}{j\omega\rho_w}P_w(\mathbf{x})\right\}$
Material state	$\{\rho_w, \kappa_w\}$	$\{\rho_w, \kappa_w\}$
Source state	$\{\hat{q}^S(s)\delta(\mathbf{x} - \mathbf{x}^S), 0\}$	$\{0, 0\}$

Domain $D' = (\mathbf{x} \in R^3| -\infty < x_1, x_2 < \infty, 0 < x_3 < x_3^R)$

$$\int_{(x_1, x_2)\in R^2} \exp(j\omega p_1 x_1 + j\omega p_2 x_2) \times \sin(\omega\Gamma_w x_3^R)\hat{v}_3(x_1, x_2, 0|\mathbf{x}^S)\, dA + \int_{(x_1, x_2)\in R^2} \exp(jwp_1 x_1 + j\omega p_2 x_2) \times \frac{\Gamma_w}{j\rho_w}\hat{p}(x_1, x_2, x_3^R|x^S)\, dA = \begin{cases} -\hat{q}^S P_w(\mathbf{x}^S), & 0 < x_3^S < x_3^R, \\ 0, & x_3^R < x_3^S < x_{3,\,min} \end{cases} \qquad (11)$$

Using the definitions of the Radon transforms, we directly obtain the relation

$$\frac{\bar{v}_3(p_1, p_2, 0|\mathbf{x}^S)}{\hat{q}^S} = \frac{-\Gamma_w}{\sin(\omega\Gamma_w x_3^R)}\frac{\bar{p}(p_1, p_2, x_3^R|\mathbf{x}^S)}{j\rho_w\hat{q}^S} - \begin{cases} \dfrac{P_w(\mathbf{x}^S)}{\sin(\omega\Gamma_w x_3^R)}, & 0 < x_3^S < x_3^R, \\ 0, & x_3^R < x_3^S < x_{3,\,min}. \end{cases} \qquad (12)$$

In the next section, the pressure wavefield is split into an incident wavefield component and a scattered wavefield component, to derive an integral equation in which only the scattered pressure wavefield of the actual state and the reflected wavefield of the desired state occur.

ACTUAL MULTIPLE REMOVAL PROCEDURE

Our purpose is to derive an integral equation for the desired pressure wavefield in which only the scattered pressure wavefield of the actual state,

$$\hat{p}^{sct} = \hat{p} - \hat{p}^{inc,H}, \qquad (13)$$

and the reflected wavefield quantities of the desired state,

$$\hat{p}^r = \hat{p}^d - \hat{p}^{inc}, \qquad (14)$$

occur. The incident pressure wavefield $\hat{p}^{inc}$ of the desired state is given by

$$\hat{p}^{inc}(\mathbf{x}^R|\mathbf{x}^S) = j\omega\rho_w\hat{q}^S\frac{\exp\left(-j\dfrac{\omega}{c}|\mathbf{x}^R - \mathbf{x}^S|\right)}{4\pi|\mathbf{x}^R - \mathbf{x}^S|}. \qquad (15)$$

The equivalent expressions for $\bar{p}^{inc}$ and $\bar{p}^{inc}$ in the Radon domain are given by

$$\bar{p}^{inc}(p_1, p_2, x_3^R|\mathbf{x}^S) = j\omega\rho_w\hat{q}^S\frac{\exp(j\omega p_1 x_1^S + j\omega p_2 x_2^S - j\omega\Gamma_w|x_3^R - x_3^S|)}{2j\omega\Gamma_w} \qquad (16)$$

and

$$\bar{p}^{inc}(\mathbf{x}^R|p_1, p_2, x_3^S) = j\omega\rho_w\hat{q}^S\frac{\exp(-j\omega p_1 x_1^R - j\omega p_2 x_2^R - j\omega\Gamma_w|x_3^R - x_3^S|)}{2j\omega\Gamma_w}. \qquad (17)$$

The incident wavefield $\hat{p}^{inc,H}$ of the actual state is given by

$$\hat{p}^{inc,H}(\mathbf{x}^R|x_1^S, x_2^S, x_3^S) = \hat{p}^{inc}(\mathbf{x}^R|x_1^S, x_2^S, x_3^S) - \hat{p}^{inc}(\mathbf{x}^R|x_1^S, x_2^S, -x_3^S). \quad (18)$$

The equivalent expressions for $\tilde{p}^{inc,H}$ and $\tilde{p}^{inc,H}$ follow directly from equations (15)–(18). After substitution of equations (13) and (14) into integral equation (9), we arrive at

$$\hat{q}^S(\omega)\hat{p}^r(\mathbf{x}^R|\mathbf{x}^S) = \hat{q}^S(\omega)[\hat{p}^{inc,H}(\mathbf{x}^R|\mathbf{x}^S) - \hat{p}^{inc}(\mathbf{x}^R|\mathbf{x}^S) + \hat{p}^{sct}(\mathbf{x}^R|\mathbf{x}^S)] - \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^{inc}(\mathbf{x}^R|p_1, p_2, 0) \times \tilde{v}_3(p_1, p_2, 0|\mathbf{x}^S)\, dA - \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^r(\mathbf{x}^R|p_1, p_2, 0) \times \tilde{v}_3(p_1, p_2, 0|\mathbf{x}^S)\, dA. \quad (19)$$

Next, we evaluate the first integral on the right-hand side of equation (19) for the case $(0 < x_3^R < x_3^S)$. Substituting the expression for the particle velocity [see equation (12)] and using equation (13), we arrive at two constituents. The integrands of the two constituents contain the multiplications of $\tilde{p}^{inc}$ with $\tilde{p}^{inc,H}$ and $\tilde{p}^{inc}$ with $\tilde{p}^{sct}$, respectively. Substituting the expressions for the incident fields in the first constituent, this integral becomes

$$\left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^{inc}(\mathbf{x}^R|p_1, p_2, 0) \times \frac{-\Gamma_w}{j\rho_w \sin(\omega\Gamma_w x_3^R)} \tilde{p}^{inc,H} \times (p_1, p_2, x_3^R|\mathbf{x}^S)\, dA = \hat{q}^S[-]. \quad (20)$$

Equation (20) can be recognized as a contribution representing the primary water surface reflection. Subtraction of this integral leads to the removal of this wavefield constituent. After substitution of the incident field in the second constituent, we arrive at

$$\left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^{inc}(\mathbf{x}^R|p_1, p_2, 0) \times \frac{-\Gamma_w}{j\rho_w \sin(\omega\Gamma_w x_3^R)} \tilde{p}^{sct}(-)\, dA = \hat{q}^S[\hat{p}^{down}(x^R|x^S). \quad (21)$$

This expression is a representation for the downgoing pressure wavefield measured at $\mathbf{x}^R$ due to a point source positioned at $\mathbf{x}^S$ (see Fokkema and Van den Berg, 1993, Chap. 11). Therefore, after subtraction of the downgoing pressure wavefield from the total scattered pressure wavefield, the upgoing pressure wavefield remains. This upgoing pressure wavefield is given by

$$\hat{p}^{up}(x_1, x_2, x_3|\mathbf{x}^S) = \hat{p}^{sct}(x_1, x_2, x_3|\mathbf{x}^S) - \hat{p}^{down}(x_1, x_2, x_3|\mathbf{x}^S) = F_{-1}^R\left\{\frac{\exp(j\omega\Gamma_w x_3)}{2j\sin(\omega\Gamma_w x_3^R)} F^R\{\hat{p}^{sct}(x_1, x_2, x_3|\mathbf{x}^S)\}\right\}. \quad (22)$$

It is easily verified that equations (20)–(22) also hold for the case $(0 < x_3^S < x_3^R)$.

The second integral in the right-hand side of equation (19) is evaluated, again for the case $(0 < x_3^R < x_3^S)$. After substituting the expression for the particle velocity (see equation (12)) and using equation (13), we arrive at two constituents. The integrands of the two constituents contain the multiplications of $\tilde{p}^r$ with $\tilde{p}^{inc,H}$ and $\tilde{p}^r$ with $\tilde{p}^{sct}$, respectively. Substituting the expression for the incident field in the first constituent, this integral becomes

$$\left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^r(\mathbf{x}^R|p_1, p_2, 0) \frac{\Gamma_w}{j\rho_w \sin(\omega\Gamma_w x_3^R)} \times \tilde{p}^{inc,H}(p_1, p_2, x_3^R|\mathbf{x}^S)\, dA = \hat{q}^S\tilde{p}^r(\mathbf{x}^R|p_1, p_2, -x_3^S). \quad (23)$$

Note that equation (23) is the desired reflected pressure wavefield due to a source located at the image point of $\mathbf{x}^S$ with respect to the reflecting surface at $x_3 = 0$. Again, the same expression holds for the case $(0 < x_3^S < x_3^R)$.

Substituting the results of equations (20)–(23) in equation (19), we arrive at

$$\hat{p}^r(\mathbf{x}^R|x_1^S, x_2^S, x_3^S) - \hat{p}^r(\mathbf{x}^R|x_1^S, x_2^S, -x_3^S) = \hat{p}^{up}(\mathbf{x}^R|\mathbf{x}^S) + \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^r(\mathbf{x}^R|p_1, p_2, 0) \times \frac{\Gamma_w}{j\rho_w \sin(-)} \frac{\tilde{p}^{sct}(-)}{\hat{q}^S}\, dA. \quad (24)$$

After extrapolating to $x_3^R = 0$ and substituting expression (22) for the upgoing pressure wavefield in terms of the scattered pressure wavefield, equation (24) becomes

$$\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, x_3^S) - \hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, -x_3^S) = \hat{p}^{up}(x_1^R, x_2^R, 0|\mathbf{x}^S) + \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^r(x_1^R, x_2^R, 0|p_1, p_2, 0) \times \frac{2\Gamma_w}{\rho_w} \frac{\tilde{p}^{up}(p_1, p_2, 0|\mathbf{x}^S)}{\hat{q}^S}\, dA. \quad (25)$$

To align the vertical source positions of the desired reflected pressure wavefield, we apply a Radon transform with respect to the source positions to the left-hand side of equation (25), with the result that

$$\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, x_3^S) - \hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, -x_3^S) = F_{-1}^S\{2j\sin(\omega\Gamma_w x_3^S) F^S\{\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0)\}\}. \quad (26)$$

By substituting this result in equation (25), applying a Radon transform with respect to the source coordinates, dividing by $2j \sin(\omega\Gamma_w x_3^S)$, and applying an inverse Radon transform, we arrive at

$$\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) = \hat{p}^{deg}(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) - \int_{(x_1,x_2)\in R^2} \hat{p}^r(x_1^R, x_2^R, 0|x_1, x_2, 0)\hat{K}(x_1, x_2|x_1^S, x_2^S)\, dA, \quad (27)$$

in which $\hat{p}^{deg}$ denotes the deghosted scattered pressure wavefield, given by

$$\hat{p}^{deg}(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) = F_{-1}^S\left\{\frac{1}{2j\sin(\omega\Gamma_w x_3^S)} F^S\{\hat{p}^{up}(x_1^R, x_2^R, 0|x_1^S, x_2^S, x_3^S)\}\right\}. \quad (28)$$

The kernel $\hat{K}$ is obtained from the inverse Fourier transform of the Radon type, with respect to the receiver slownesses as

$$\bar{K}(p_1, p_2|x_1^S, x_2^S) = \frac{-2\Gamma_w}{\hat{q}^S\rho_w}\bar{p}^{deg}(p_1, p_2, 0|x_1^S, x_2^S, 0). \quad (29)$$

Equation (27) is a linear integral equation of the second kind. This can be written as an operator equation of the form

$$\mathbf{\hat{a}} + \mathbf{\hat{K}}\mathbf{\hat{a}} = \mathbf{\hat{b}}, \quad (30)$$

in which $\mathbf{\hat{b}}$ denotes the deghosted scattered pressure wavefield, $\mathbf{\hat{a}}$ denotes the reflected pressure wavefield, and $\mathbf{\hat{K}}$ denotes the kernel. Note that the integral variables in integral equation (27) are the horizontal coordinates of the various source locations of the desired wavefield while the receiver position is fixed. Hence, the solution is obtained in the common-receiver domain. Note also that the kernel and the known term of the integral equation are both expressed in terms of the deghosted pressure wavefield. In view of equation (29), the kernel of the integral equation is nonsingular. Therefore, any integration rule can be used to replace the integration by a discrete summation. This procedure leads to a system of linear algebraic equations for the discrete values of the desired reflected pressure wavefield, which can be solved numerically. An alternative way to solve the integral equation is based on a Neumann series

$$\mathbf{\hat{a}} = \mathbf{\hat{b}} - \mathbf{\hat{K}}\mathbf{\hat{b}} + \mathbf{\hat{K}}(\mathbf{\hat{K}}\mathbf{\hat{b}}) - \cdots + (-\mathbf{\hat{K}})^n\mathbf{\hat{b}} + \cdots, \quad (31)$$

provided that its convergence can be proved. This topic is discussed in the next section.

THE NEUMANN SERIES

We start with transforming integral equation (27) back to the time domain, leading to

$$p^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, \tau) = \chi_{T_1}(t)p^{deg}(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t) - \int_{(x_1,x_2)\in R^2} dA \times \int_{\tau=t_1}^{t} p^r(x_1^R, x_2^R, 0|x_1, x_2, 0, t-\tau) \times K(x_1, x_2|x_1^S, x_2^S, \tau)\, d\tau, \quad (32)$$

in which $\chi_{T_1}(t)$ denotes the unit-step function for time interval T_1. Time interval T_1 is determined by noting that the deghosted field has traveled at least from plane $x_3 = 0$ to and from the plane $x_3 = x_{3,min}$. When we assume that the seismic experiment starts at $t = 0$, then, in view of causality, we have

$$T_1 = \{t \in R, t > t_1 = 2x_{3,min}/c_w\}. \quad (33)$$

Since the kernel depends only on the deghosted pressure wavefield, we conclude that the same holds for the kernel. Therefore, the lower bound of the integration of the convolution is set at $\tau = t_1$. The upper bound follows directly from the causality of the reflected desired pressure wavefield. After applying a Neumann series to equation (32), we arrive at

$$p^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t) = \sum_{n=0}^{\infty} p_n^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t), \quad (34)$$

in which the first term is given by

$$p_0^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t) = \chi_{T_1}(t)p^{deg}(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t), \quad (35)$$

and the nth term follows from the $(n-1)$th as

$$p_n^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t) = -\int_{(x_1,x_2)\in R^2} dA \int_{\tau=t_1}^{t} p_{n-1}^r(x_1^R, x_2^R, 0|x_1, x_2, 0, t-\tau) \times K(x_1, x_2|x_1^S, x_2^S, \tau)\, d\tau. \quad (36)$$

Next, we prove by induction that p_n^r is zero for $t < (n+1)t_1$. Inspecting equation (35), we observe that this is true for the first term $n = 0$. Let us now assume that p_{n-1}^r is zero for $t < nt_1$. It follows that $p_{n-1}^r(x_1^R, x_2^R, 0|x_1, x_2, 0, t-\tau)$ vanishes for $t - \tau < nt_1$. Since the integration variable $\tau > t_1$, we conclude that $p_{n-1}^r(x_1^R, x_2^R, 0|x_1, x_2, 0, t-\tau)$ vanishes for $t < (n+1)t_1$. As a consequence, we conclude from equation (36) that $p_n^r(x_1^R, x_2^R, 0|x_1, x_2, 0, t)$ vanishes for $t < (n+1)t_1$. Therefore, we may write

$$p^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t) = \sum_{n=0}^{\infty} \chi_{T_{n+1}}(t)p_n^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t), \quad (37)$$

in which

$$T_{n+1} = \{t \in R, t > t_{n+1} = (n+1)t_1 = 2(n+1)x_{3,min}/c_w\}. \quad (38)$$

For a finite time interval of observation, $0 < t < t_{N+1}$, the summation is confined to $0 \leq n \leq N-1$, and the Neumann series is convergent. Therefore, we replace the infinite summation by a finite one

$$p^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t) = \sum_{n=0}^{N-1} p_n^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0, t), \quad (39)$$

$$\text{for} \quad 0 < t < t_{N+1} = 2(N+1)x_{3,min}/c_w.$$

Computing the desired pressure wavefield in this manner leads to a successive removal of the water surface multiples, such that each higher-order iteration removes a higher-order multiple. The first-order term has already been identified as the deghosted field. The second-order term eliminates all contributions from the actual wavefield that have been reflected once against the water surface (removal of first-order multiples). Similarly, the $(n + 1)$th-order term removes the nth-order water surface multiples.

The temporal convolution in equation (36) is equivalent to an algebraic multiplication in the frequency domain. Transforming equations (36) and (39) to the frequency domain we arrive at

$$\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) = \sum_{n=0}^{N-1} \hat{p}_n^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0), \qquad (40)$$

and

$$\hat{p}_n^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) = -\int_{(x_1,x_2)\in R^2} \hat{p}_{n-1}^r(x_1^R, x_2^R, 0|x_1, x_2, 0)\hat{K}(x_1, x_2|x_1^S, x_2^S)\, dA. \qquad (41)$$

It is noted that we cannot conclude that equation (41) is convergent for every frequency when $N \rightarrow \infty$, since the convergence of the Neumann series has been proven only in the time domain. Hence, the results of the Neumann series are only useful in the time domain within a finite time interval.

INTEGRAL EQUATION IN THE DOUBLE RADON DOMAIN

In the previous sections, an integral equation was derived for the desired reflected wavefield. The integral variables in the integral equation are the horizontal coordinates of the various source locations of the desired wavefield while the receiver position is fixed. Hence, the solution is obtained in the common-receiver domain. An alternative domain in which to solve the integral equation is the double Radon domain, where both source and receiver coordinates are transformed to the slowness domain. We start with the integral equation [see equation (27)]

$$\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) = \hat{p}^{deg}(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) - \int_{(x_1,x_2)\in R^2} \hat{p}^r(x_1^R, x_2^R, 0|x_1, x_2, 0)\hat{K}(x_1, x_2|x_1^S, x_2^S)\, dA. \qquad (42)$$

Substituting for kernel $\hat{K}$, defined in equation (29), its double Radon transformed equivalence $\tilde{\hat{K}}$, we get

$$\hat{p}^r(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) = \hat{p}^{deg}(x_1^R, x_2^R, 0|x_1^S, x_2^S, 0) - \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1^S,p_2^S)\in R^2} \exp(j\omega p_1^S x_1^S + j\omega p_2^S x_2^S)\, dA \times \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{p}^r(x_1^R, x_2^R, 0|p_1, p_2, 0) \times \tilde{\hat{K}}(p_1, p_2|p_1^S, p_2^S)\, dA. \qquad (43)$$

Next, we apply the forward Radon transform with respect to both source and receiver coordinates to equation (43). We then arrive at

$$\tilde{\tilde{p}}^r(p_1^R, p_2^R, 0|p_1^S, p_2^S, 0) = \tilde{\tilde{p}}^{deg}(p_1^R, p_2^R, 0|p_1^S, p_2^S, 0) - \left(\frac{\omega}{2\pi}\right)^2 \int_{(p_1,p_2)\in R^2} \tilde{\tilde{p}}^r(p_1^R, p_2^R, 0|p_1, p_2, 0) \times \tilde{\tilde{K}}(p_1, p_2|p_1^S, p_2^S)\, dA, \qquad (44)$$

with

$$\tilde{\tilde{K}}(p_1, p_2|p_1^S, p_2^S) = \frac{-2\Gamma_w}{\hat{q}^S \rho_w}\, \tilde{\tilde{p}}^{deg}(p_1, p_2, 0|p_1^S, p_2^S, 0), \qquad (45)$$

in which $\tilde{\tilde{p}}^{deg}$ is the deghosted acoustic pressure field in the double Radon domain.

Equation (44) is an integral equation of the second kind. We note that the integral variables are the horizontal receiver slowness positions of the kernel and the horizontal shot slowness positions of the desired reflected wavefield. This means that after solving the integral equation, the desired reflected wavefield has been calculated for a fixed receiver slowness value for all relevant source slowness values. To obtain the result in the space-time domain, the integral equation must be solved for *all* receiver slownesses.

It is easily verified that for a horizontally layered earth model the kernel becomes a diagonal matrix. In general, for a laterally varying earth, the data in the kernel is mapped around the diagonal of the matrix. This fact can be used to speed the calculation significantly, since fast numerical routines can be used to solve the integral equation.

Again, the kernel of the integral equation is nonsingular. Therefore, the integral equation can be discretized and solved, either by matrix inversion or by a Neumann series. After the removal of surface-related wave phenomena, the data are still in the double Radon domain. To obtain the result in the space-domain, an inverse double Radon transform must be applied to the data. It is noted that some processing techniques operate in the double Radon domain and inversion to the spatial domain is superfluous.

NUMERICAL TESTS AND RESULTS

To test the multiple removal scheme, a 2-D synthetic data set from a rigid strip embedded in a semi-infinite water layer has been computed. The computer implementation of the forward problem is based on a conjugate-gradient iterative solution of an integral equation over the strip domain (Van den Berg, 1984). The rigid strip is 140 m wide at a depth of 100 m. The source depth x_3^S is 7.5 m and the receiver depth x_3^R is 5 m. We processed 501 shots with 181 traces per shot in a split-spread configuration, with 512 time samples per trace. Shot and receiver spacing were 3.5 m. Figure 5a shows the center common-shot gather after removal of the direct wave and its water surface reflection. The primary reflection, the source/receiver ghosts, the water surface multiples, and the diffracted energy caused by the edges from the strip are clearly distinguishable. It is noted that in this configuration, no internal multiples occur. The ideal output for the center position is shown in Figure 5b. In Figure 5c, the deghosted center shot record is shown. To illustrate the data distribution

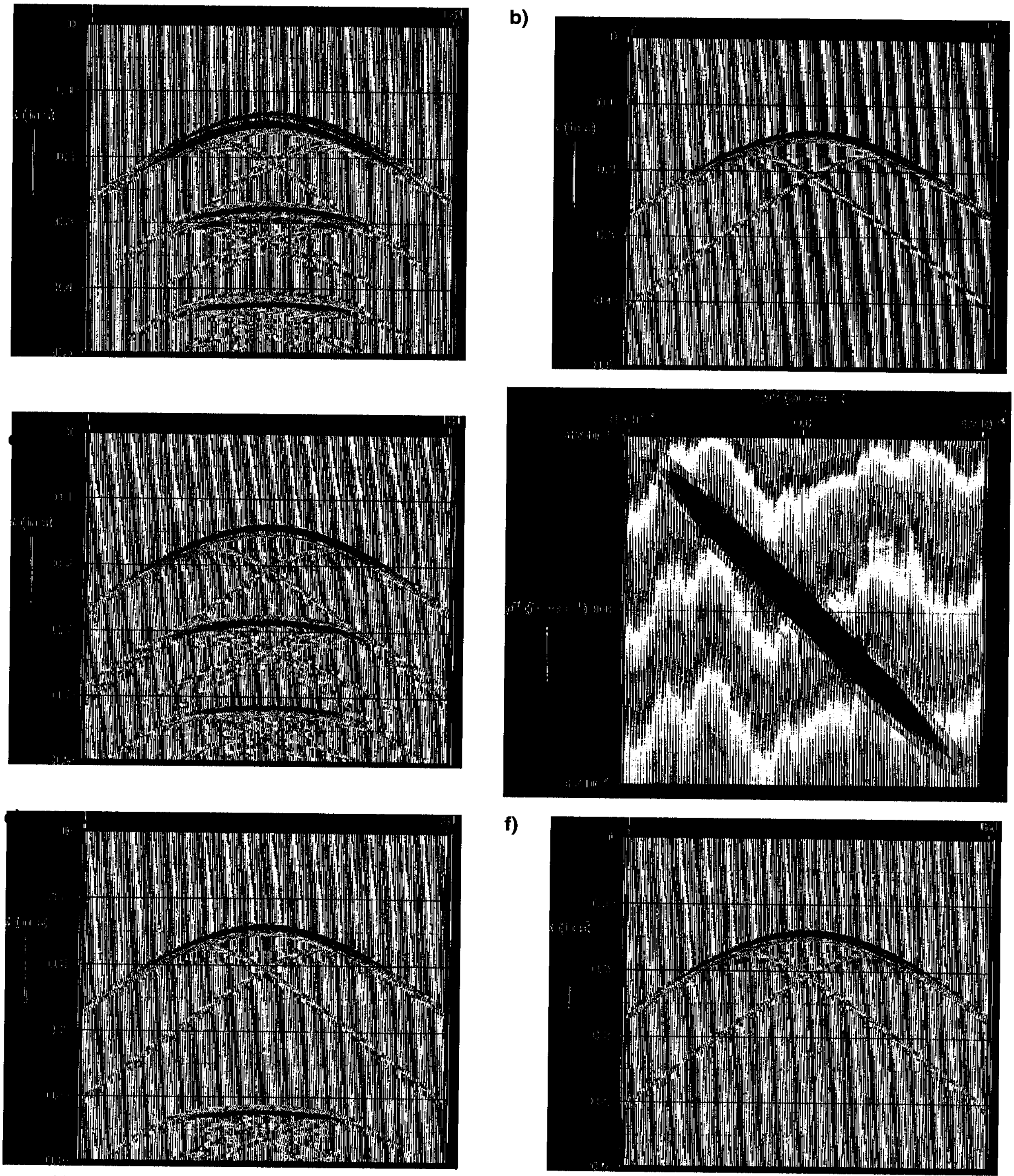

FIG. 5. (a) Input shot gather for the center position above the strip. (b) Ideal output for the multiple removal scheme for the center position above the strip. (c) Deghosted shot gather. (d) Data distribution in the kernel in the $p^R - p^S - \tau$-domain after stacking over all τ values. (e) Output after removal of the first-order multiple. (f) Output after removal of the second-order multiple.

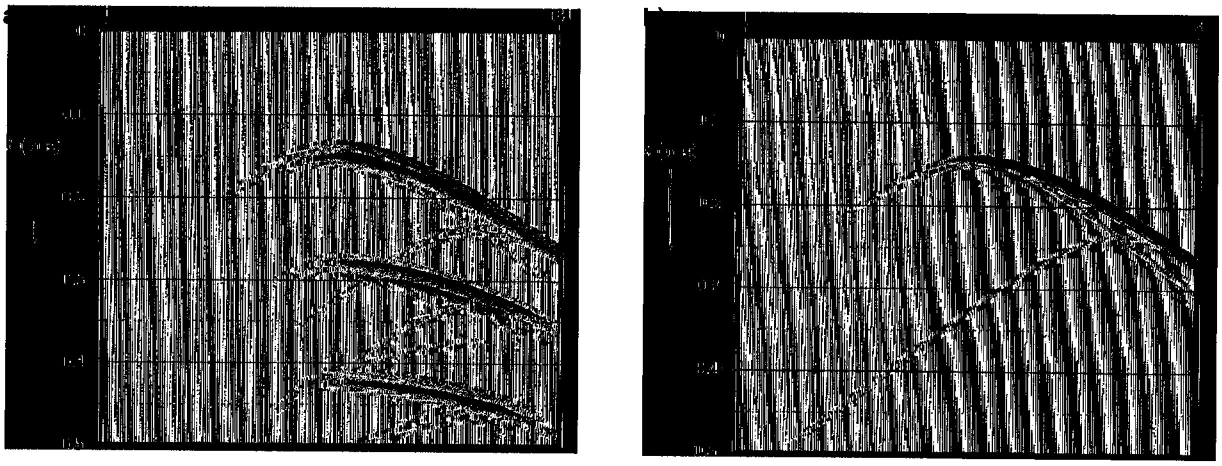

FIG. 6. (a) Input shot gather for the position above the left-hand edge of the strip. (b) Output after multiple removal using matrix inversion for the position above the left-hand edge of the strip.

in the kernel, the data are transformed back to the $p^R - p^S - \tau$-domain and stacked over all τ-values. The result is shown in Figure 5d. It is noted that the seismic energy is centered around the diagonal, which enables the use of fast numerical routines to perform the matrix inversion and the Neumann series. The Neumann series solution of the integral equation is shown in Figures 5e and 5f. Figure 5e shows the result after removal of the first-order multiples; Figure 5f illustrates the result after removal of the second-order multiples. Note the excellent agreement with the ideal situation, where the diffracted energy from the edges of the strip is perfectly preserved. For this data set, the Neumann series solution is computationally faster when the data contain less than three surface-related multiples. The use of the matrix-inversion method becomes more favorable when the data are contaminated with more than two multiples.

Figure 6a shows the input shot gather for the position above the left-hand edge of the strip. The matrix inversion solution of the integral equation is shown in Figure 6b.

CONCLUSIONS

Using Rayleigh's reciprocity theorem, a scheme is derived to remove surface-related wave phenomena. No information about the subsurface geology, neither material nor structural, is required to remove the effects of the free surface. Application of Rayleigh's reciprocity relation leads to an integral equation of the second kind. Two domains are considered to solve the integral equation: the space domain and the double Radon domain. The advantage of solving the integral equation in the double Radon domain is that it reduces the computation time significantly since the data in the kernel are mapped around the diagonal. Therefore, fast numerical procedures can be applied to solve the integral equation. We considered two solution methods for the integral equation: matrix inversion and a Neumann series. Matrix inversion removes all multiples at once; whereas, the Neumann series leads to a successive removal of the water surface multiples such that each higher-order term removes a higher order multiple. The method was tested on an independently calculated data set, a laterally varying rigid strip model. Excellent results were obtained for this data set. Future research will concentrate on estimating the wavelet from the data and on handling 2-D data in a 3-D world.

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